Subcarrier Pairing and Relay Assignment With Improved User Fairness in a Multi-user Cooperative OFDM System

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***Abstract*—We consider a joint relay selection and subcarrier allocation problem that minimizes the total system power for a multi-user, multi-relay and single source cooperative OFDM based two hop system. The system is constrained to all users having a specific subcarrier requirement (user fairness). However no specific fairness constraints for relays are considered. To ensure the optimum power allocation, the subcarriers in two hops are paired with each other. We obtain an optimal subcarrier allocation for the single user case using a similar method to what is described in [1] and modify the algorithm for multiuser scenario. Although the optimality is not achieved in multiuser case the probability of all users being served fairly is improved significantly with a relatively low cost trade off.**

I. Introduction

Recently subcarrier pairing has become a highlighted research area in two-hop Orthogonal Frequency Division Multiplexing (OFDM) based communication systems [1–3]. Pairing simply refers to the coupling of a particular subcarrier in the first hop with another in the second hop. Then the information sent from source to relay through first subcarrier can only be resent to destination through it's pair. An OFDM system having multiple subcarriers offers per subcarrierwise resource allocation options that makes pairing possible. It has been shown that in a two hop communication, pairing of subcarriers improves the system performance [4–6]. Therefore which subcarriers should be paired to be used with a particular source and a relay is an interesting resource allocation problem.

In a multi-hop communication system this allocation process is not simple. By looking at the first hop channel it might seem that allocating a particular subcarrier to a user, relay pair is beneficial if that subcarrier has a high channel gain. However all channels to the intended destination from that selected relay could have low gains and therefore allocating resources to that user, relay pair would not be efficient. Therefore it is apparent that an allocation decision of resources should be taken by looking at the channels of both hops with respect to the intermediate relay. Inarguably it is clear that intelligent allocation decisions could satisfy low power and/or high rate requirements more efficiently.

Simplest form of pairing would be to pair a subcarrier in the first hop with the same subcarrier in the second hop. This is a conventional, inefficient method of pairing. A more efficient subcarrier pairing method in a single user and single relay system is studied in [4, 5]. Authors claim that simple ranking of subchannels in both hops and then pairing high ranked channels together leads to a capacity maximized optimal solution in a frequency selective channel. A Multiple Input Multiple Output-OFDM (MIMO-OFDM) system with single source relay and destination is considered in [6]. Here also it is suggested that pairing will improve the performance and that a strong subcarrier in the first hop should be coupled with a strong subcarrier in the second hop. A subcarrier paring scheme using Lloyd's algorithm for a system with single source, relay and destination is studied in [3]. All the above mentioned research has focused on systems that has a single source, a single relay and a single destination.

Reference [7] considers a system where multiple sources are transmitting to a destination via multiple relays. However authors only study a fair relay assignment scheme and same subcarrier is paired in both hops. Subcarrier pairing in a relay aided cognitive radio network is investigated in [2]. Here a licensed primary user shares the bandwidth with an unlicensed secondary user with no specific constraints being given for each user to use the available subcarriers. In [1] an optimal channel and relay assignment algorithm for subcarrier pairing is proposed. In this allocation algorithm for each possible subcarrier pairings, the cost associated with using a particular relay for a user, destination (source pair) is calculated. Then for each subcarrier pairing the minimum cost source pair is selected and optimum pairings from these selection is chosen by maximum weighted bipartite mapping. While this algorithm is optimal authors have not included a fairness constraint for users. Therefore in optimal solution some users might not have any subcarrier pair assigned.

In a practical scenario all users should be allocated resources. In this paper we modify the above algorithm in [1] to increase the probability of all users being catered. When a fairness constraint is included the algorithm looses the optimality. It can be seen that in order to cater for users fairly, the resources must be sacrificed by means of cost. If an equally fair system is to be employed then the complexity of

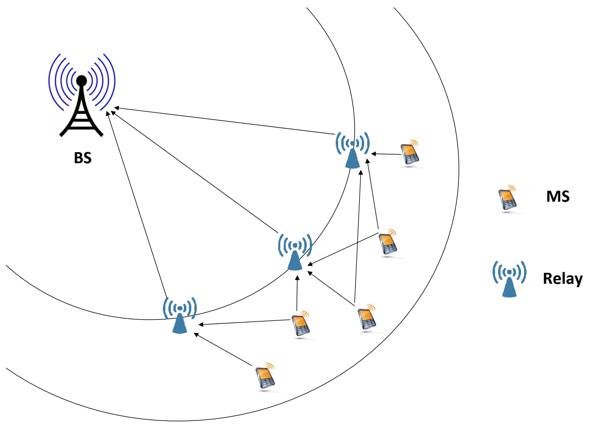

Fig. 1: System model.

the algorithm increases while the total cost of the allocation moves from optimal solution. Therefore we propose an intermediate heuristic algorithm where, the algorithm increases the probability of all users being assigned subcarrier pairs while not necessarily ensuring that a specific user requirement is met. Additionally this heuristic approach does not improve the complexity of the algorithm by that much while operating close to the optimal solution.

II. System Model

We consider an OFDM based multi-user multi-relay system. There are K mobile stations (MS) in the system that want to communicate with the Base Station (BS) via R relays. Both Relays and BS are stationary. We assume that no mobile can directly access BS while all relays are in the range of BS. Also mobile stations participating in transmission are scattered relatively evenly so that all relays can service them. N number of subcarriers are paired for MS to relay and relay to BS communication. Each subcarrier is assumed to be subject to flat fading and the channel is assumed to be varying very slowly over the time. We further assume that compared to mobile stations relays have additional power, so that any one relay can employ all the subcarriers if necessary in a particular time slot without draining its power. Furthermore we consider the high Signal to Noise Ratio (SNR) region of the transmission.

A. Communication Protocol

MS requests aid from relays via a predetermined channel. Idle relays in range will agree to cooperate as relays. Each relay will then estimate the channel to MS and BS using a training sequence. Relays will cooperate with each other and decide on the allocation criteria based on the proposed algorithm. When the allocation is done and decisions are cooperated in between relays, they will signal the MS's to start transmission. When a relay receives a message, it will decode the subcarriers intended for it and will retransmit those messages to BS via allocated subcarriers for the second hop. Relays will also indicate to BS the specific subcarrier assignment for each user. BS will decode the message accordingly.

B. Problem Formulation

If the power required at the receiver to correctly decode a message via particular subcarrier is P then the objective is to find subcarrier allocation $a_{k,r}^{n,n'}$ that minimizes the total power P_T.

$$minimize\ P_T = \sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{n=1}^{N}\sum_{n'=1}^{N} a_{k,r}^{n,n'} \left\{ \frac{P}{h_n^{k,r}} + \frac{P}{h_{n'}^{r}} \right\} \quad (1)$$

where $a_{k,r}^{n,n'} = \{0, 1\}$. The notation denotes whether the subcarrier pair (n, n') is allocated or not to user k and relay r. $h_n^{k,r}$ denotes the channel gain via subcarrier n over user k and relay r, while $h_{n'}^{r}$ denotes channel over subcarrier n' over relay r and the BS.

To avoid inter carrier interference, it should be assured that no pair can be used by more than one user. Therefore above minimization is constrained to,

$$\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{n=1}^{N} a_{k,r}^{n,n'} \leq 1 \ \forall \ n' \quad (2)$$

$$\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{n'=1}^{N} a_{k,r}^{n,n'} \leq 1 \ \forall \ n \quad (3)$$

In order to ensure fair assignments a fairness constraint needs to be introduced.

$$\sum_{r=1}^{R}\sum_{n'=1}^{N}\sum_{n=1}^{N} a_{k,r}^{n,n'} = S_k \ \forall \ k \quad (4)$$

S_k denotes the minimum subcarrier requirement for user k. Note that no constraints for the number of subcarriers that can be serviced by a relay is considered. This requires that in a worse case a single relay must be prepared to supply power for all subcarriers. Such a case might result in that particular relay being drained out of power. However we assume that users are separated from each other by a fair distance so that no single relay is catering a large number of users at a given time. Also since relay station is not mobile it is fair to assume that it has excess power compared to an MS, which is the case in practice.

Above optimization problem is NP-hard and is difficult to track in real time when N, K, R are large. Therefore we propose a heuristic algorithm to solve this problem.

III. Allocation Algorithm

A. Single User case

First we consider a single user case. For a single user case fairness constraint (4) need not be considered. Although the problem formulation is different, a solution can be achieved by using a similar algorithm as described in [1]. We formulate the square *Cost Matrix* (A matrix that holds a particular cost

$$
\begin{pmatrix}
\boxed{P_{1,1}^{n,n'}} & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
P_{K,1}^{n,n'} & \cdots & \cdots & P_{K,R}^{n,n'}
\end{pmatrix}
\begin{pmatrix}
P_{1,1}^{n,m} & \cdots & \cdots & \cdots \\
\cdots & \boxed{P_{2,2}^{n,m}} & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
P_{K,1}^{n,m} & \cdots & \cdots & P_{K,R}^{n,m}
\end{pmatrix}
$$

Fig. 2: Tow sub matrices for subcarrier pairs (n, n') and (n, m). A possible selection criteria is highlighted for a case where $S_1 = 1$ and $P_{2,2}^{n,m} > P_{1,1}^{n,m}$.

value assigned for each subcarrier pairings) for all possible subcarrier pairings using $P_{k^*,r^*}^{n,n'}$ where,

$$P_{k^*,r^*}^{n,n'} = min(P_{k,r}^{n,n'}) \quad \forall\, k, r \tag{5}$$

$P_{k,r}^{n,n'}$ denotes the total power required for transmitting a message correctly if subcarriers n and n' were paired and allocated to user k and relay r. k^* and r^* in turn then denotes the user, relay pair for minimum power cost for (n, n') pairing. Now each value in cost matrix corresponds to a particular subcarrier pairing (n, n') and also represents a specific user and a relay for this pairing. Our requirement is to select the pairs that result in total minimum cost while no subcarrier in first hop is paired with a subcarrier in the second hop more than once. It is clear that this is a standard assignment problem that can be solved using the Hungarian Method [8]. Since the the fairness constraint in (4) can be ignored the above solution is optimum for the single user case.

B. Multi User case

Selection criteria used in Equation (5) essentially damages the fairness constraint in (4). Here we propose a suboptimal algorithm that does not directly satisfy the fairness constraint but indirectly increase the probability of all users being fairly served.

We modify the algorithm as follows. Our objective is to minimize the power required to achieve a given rate for all users. Initially a cost matrix is formed that contains all the power costs over all possible pairings. For all (n, n') pairs, cost matrix contains k by r sub matrices. Then for a particular user, if a particular subcarrier in the first hop gives minimum power cost that pairing is selected. However for the same user, with the same subcarrier in the first hop, the chance to pair with a different subcarrier in the second hop is only given if that user has been allocated less than S_k pairs. Otherwise with same first hop subcarrier, user is not given a chance to pair with another second hop subcarrier. Using these selections modified cost matrix is formed.

Two sub matrices for pairings (n, n') and (n, m), (note that $n' < m$) is shown in Fig. 2 and a possible selection is marked in red. Assume that S_1 is equal to one and $P_{2,2}^{n,m} > P_{1,1}^{n,m}$. Since user one is allowed to pair with (n, n'), it will not be given a chance again to use n to pair with another subcarrier. Therefore (n, m) pairing is allocated to second user.

Parameter	**Value/Behaviour**
No. of Subcarriers	256
No.of Relays	3
Modulation	2-QAM
SER	10^{-4}
Channel Model	Rayleigh fading, AWGN
Channel Behavior	Slow, Frequency selective
No.of Multipaths	4

TABLE I: Simulation Parameters

This selection scheme essentially ensures that in the next stage of algorithm modified cost matrix represents each users fairly. Once this selection is done, cost matrix contains N by N values with each user being represented S_k times. Then Hungarian algorithm can be used to find the minimum cost allocation from the cost matrix. While this second step does not ensure fairness, since the cost matrix already represents all users equally, initial step increases the probability of all users being served.

Proposed algorithm is summarized below.

Algorithm 1 Subcarrier Pairing

$Formulate\ Cost\ Matrix$
for $All\ subcarriers\ in\ first\ hop$ **do**
  for $All\ subcarriers\ in\ second\ hop$ **do**
    if $A\ user''s\ assigned\ subcarriers\ less\ than\ S_k$ **then**
      $Choose\ minimum\ cost\ pairing$
      $Add\ pair\ to\ modified\ cost\ matrix$
    else
      $Choose\ next\ user$
      $Add\ pair\ to\ modified\ cost\ matrix$
    end if
  end for
end for
$find\ best\ assignment\ for\ modified\ cost\ matrix$

IV. Simulation Results

The performance of the proposed algorithm is tested using Monte Carlo simulation methods in a time invariant Rayleigh fading channel with Additive White Gaussian Noise (AWGN). High SNR region is considered. The power required to transmit a message in a QAM modulation scheme for a given Symbol to Error Ratio (SER) is calculated for both hops for all subcarriers. These power values are used to formulate the cost matrix. Then using the proposed algorithm modified cost matrix is formed and best assignment is found using Hungarian method.

Fig. 3 shows a measure of the probability of a user not being served at least one subcarrier. This is plotted for a 256 subcarriers case. When the number of users increases the probability of a user not being served increases drastically in Yuan's algorithm. The reason is that when number of users increases the number of subcarriers allocated for a users

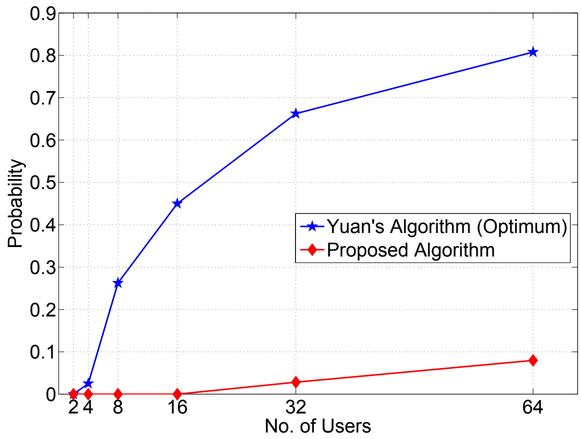

Fig. 3: Probability of a user not being assigned at least one subcarrier pair in a 256 subcarrier system.

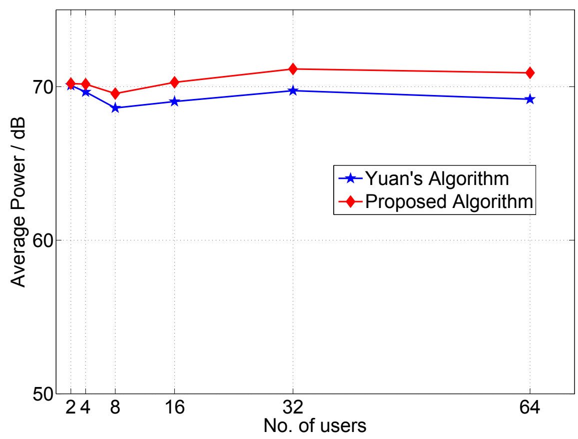

Fig. 4: Total power requirement averaged for 2-QAM modulation in a 256 subcarrier system.

decreases and subsequently the chance for a user having no good pairings at all increases.

Fig. 4 depicts the total power requirement for the system averaged for a QAM modulation scheme. 256 subcarriers are employed in the system. It can be seen that the proposed algorithm requires more power compared to the compared scheme. Note that Yuan's algorithm is optimum for the considered case. When the number of users increases proposed algorithm moves further away from the optimum position. When the number of users is 64 there is about 2dB power increase in our algorithm. But when compared with the higher probability of a user being served in the proposed algorithm, this tradeoff is relatively low.

V. Conclusion

We have proposed a suboptimal heuristic subcarrier pairing algorithm that essentially increases the probability of all users being served. However it can be seen that by changing the selection decisions of pairs from sub matrices over time any users that has not been served can also be served in time division manner. Therefore this algorithm can be modified to operate also in time domain to further improve the performance. Furthermore although we have assumed a time invariant channel, the same algorithm can be used for a time variant channel without loosing the generality. In this case if the BER goes below a pre-determined threshold the allocation algorithm needs to be run again.

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